A Remedial Intervention for Addition and Subtraction in Children with Dyscalculia

Alireza Beygi, Prakash Padakannaya, and Gowramma, I. P.
University of Mysore, Mysore

This study investigated the impact of remedial intervention on students' performance with dyscalculia in teaching addition and subtraction. Forty male students with dyscalculia (20 in experimental, and 20 in control groups) from fourth and fifth grades in Arak, Iran were the participants. The experimental group received a remedial program in addition to their regular classroom teaching (every other day). Data analysis indicated a significant increase in the subtraction and addition performance after remedial intervention. The article discusses implications for teachers, administrators, researchers, teacher training institutions, and students with learning disabilities.

**Keywords**: Dyscalculia, Addition, Subtraction, Learning Disability.

According to the DSM-IV (American Psychiatric Association, 1994) developmental dyscalculia is a rare form of learning disability (LD), with a prevalence of about 1% in the school-age population. But researches in the United States (Badian, 1983), Norway (Ostad, 1998), Israel (Gross-Tsur, Manor, & Shalev, 1996), and Europe (Kosc, 1974) have shown that 5% to 8% of school-age children exhibit some form of mathematical disabilities (MD) and associated long-term problems (Geary, 2004; Griffin & Case, 1997). With many of these students, reading disabilities (RD) and attention-deficit/hyperactivity disorder (ADHD) have been identified as co-morbid disorders (Geary, 2004; Gross-Tsur et al., 1996). Behrangi (1997 cited in Behrad, 2006) studied the prevalence of learning disabilities in primary schools (grades I, II, III, IV, and V) in Tehran, the capital city of Iran and concluded that 5.2% of the children exhibited symptoms of dyscalculia.

Children with LD often have difficulty with symbolic or abstract concepts and reasoning. These students may need extra assistance through hands-on manipulative and pictorial representations of mathematical concepts. Hands-on experiences allow students to understand numerical symbols and abstract equations at a concrete level, making the information more accessible to all students (Maccini & Gagnon, 2000). Concrete-Representational-Abstract (CRA) is an intervention for mathematics instruction that can enhance the mathematics performance of students with learning disabilities (Baroody, 1987). It is a three-part instructional strategy, with each part building on the previous instruction to promote student learning and retention and to address conceptual knowledge. The CRA sequence of instruction incorporates the use of hands-on manipulative in the concrete stage, followed by pictorial displays in the representations phase, and in the next phase facilitates abstract reasoning with numerical symbols. Learning disabled students learning basic mathematics facts with CRA instruction show improvements in acquisition and retention of mathematics concepts (Miller & Mercer, 1993). CRA supports understanding of underlying mathematical concepts before learning rules,
and facilitates children moving from a concrete model of chips or blocks to an abstract representational \((4 \times 3 = 12)\). Thus CRA instructional sequence consists of three phases. The first one is concrete phase. In the concrete phase children with learning disabilities in math are provided with manipulative and other material or physical learning tools that will provide them the opportunity to explore a mathematical concept or process by actually doing it with tools. This is the stage of ‘getting their hands dirty’ with the intent that having an actual experience will enable the construction of the knowledge being targeted.

The second phase is the representational phase. In this phase, students with learning disabilities in math were trained to develop mental images of the mathematical manipulations by drawing on other means for understanding the target knowledge. Another way to think about this phase is to say that students with learning disabilities in math are encouraged to step back form the manipulative and other concrete tools and focus on the mathematical concept or process involved in performing actions with the tools. In the abstract phase, the third level, students with learning disabilities in math could manipulate concepts or processes in the absence of the tools that were important in the early phase of learning. The concrete-representational-abstract (CRA) teaching sequence has been found to facilitate math learning in a variety of basic skills including addition (Miller, Mercer, & Dillon, 1992), place value (Peterson, Mercer, & O’Shea, 1988), subtraction (Miller & Mercer, 1992), multiplication (Morin & Miller, 1998), division (Miller, Mercer, & Dillon, 1992), and fractions (Butler, 1999). In the present study CRA approach and the lesson program developed by Gowramma (2005), were used for remedial teaching of addition and subtraction and the effectiveness of the remediation was tested with the help of a control group.

### Method

**Participants:**
Participants were selected through screening more than 1000 students studying in 4th and 5th grades in primary schools of Arak city in Iran for dyscalculia. Only 40 students who exhibited arithmetic disability were included in the present study. Their age range was 10-12 years. They were randomly divided into two groups - Experimental and Control groups (with 20 students in each group).

**Measures:**

**Key math diagnostic arithmetic test:**
This test was meant for students from kindergarten through grade eight (Cannolley et al., 1997). It covers three aspects: A) Basic concepts (numeration: 24 items, rational numbers: 18 items, geometry: 24 items). B) Operations (Addition: 18 items, Subtraction: 18 items, Multiplication: 18 items, Division: 18 items, Mental Computation: 18 items). C) Applications (Measurement: 24 items, Time and Money: 24 items, Estimation: 18 items, Problem Solving: 18 items). A correct response by the students received a score of one and a wrong response received a zero score. The students’ final scores would correspond to total correct responses. This test was used as a criterion measure in both pre and post intervention testing.

**Wechsler Intelligence Scale for Children-III:**
It is a battery of tests for 6 to 17 years old. The WISC-III consists of two sets scales, the verbal scale and the performance scale. Each of these scales has several subtests. The verbal scale measures include language expression, comprehension, and the ability to apply these skills to solving problems. The examiner presented the questions orally, and the participants gave spoken response. The performance scale assesses non-verbal problem solving, perceptual organization, and speed and visual-motor proficiency. The standard procedure of administration was followed.
**Children's Behavior Questionnaire** (Proforma B): It consists of 26 items indicating presence or absence of common behavioral and emotional problems as seen in school setting. The items describe behavior against which the teacher is asked to indicate whether each description ‘does not apply’, ‘applies to some extent’ or ‘definitely applies’ to the child. Each of these responses is scored 0, 1, and 2 respectively. The scores on each item are added together to produce a total score. The scale has been shown to have a test-retest reliability of 0.89 over a 3-month period and inter rater reliability of 0.72.

**DSM-IV (1994)** suggests 12 item check list for identifying dyscalculia (mathematics disorder), which are completed by teachers. Teachers rate students on each of the items and decide whether an item can be definitely applied or not. The ratings are 0, and 1 (0 stands for “No” and 1 stands for “Yes”). The total score is the combination of the scores of all items. Standard procedure as mentioned in the manual was followed.

**Procedure**

The study was carried out in three phases:

**Phase I**: Screening and pre-test: In the first phase, the investigator approached the primary school authorities and teachers in order to get their consent for conducting the study, and also to be directed to the students who don’t have satisfactory performance in mathematics (meaning those who don’t receive acceptable scores in mathematics). Out of the total population of 1000, their teachers selected 300 students. Finally, 40 students out of 300 were found to be having dyscalculia and were selected for this study. The investigators put them randomly into control and experimental groups (20 students in each group). All the 40 students had the following characteristics:

1- Arithmetic disability (scored 70 and below in Key math diagnostic arithmetic test, and scored above 5 in the check list of DSM IV).

2- Adequate intelligence (IQ 90 and above as assessed by WISC-III)

3- No behavioral and emotional problems (scored 9 and below through as assessed by Rutter’s Proforma B)

**Phase II**: After selection, the experimental group received remedial teaching based on CRA and Gowramma’s lesson program. Remedial teaching was done in Learning Disability Centre in Arak city, in 30 sessions (every second day). Each session was for 65 minutes. Gowramma’s teaching module consisted of six broad units: Pre –requisite skills (26 lessons), Number concept (29 lessons), Addition (25 lessons), Subtraction (23 lessons), Multiplication (23 lessons), and Division (17 lessons). In this study we employed addition and subtraction lessons only (contents of lessons are given in tables 1 and 2). Work sheets designed on the basis of CRA with pictorial representations were given to experimental group during every session.

**Table 1: Lesson scheme for teaching subtraction**

<table>
<thead>
<tr>
<th>SPECIFIC OBJECTIVES OF EACH LESSON IN SEQUENTIAL ORDER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson 1 To learn the terms and symbol of subtraction</td>
</tr>
<tr>
<td>Lesson 2 To understand that subtraction means to take away things</td>
</tr>
<tr>
<td>Lesson 3 To make familiar with all terms used in subtraction</td>
</tr>
<tr>
<td>Lesson 4 To understand the concepts of subtraction</td>
</tr>
<tr>
<td>Lesson 5 To understand the procedure of subtraction</td>
</tr>
<tr>
<td>Lesson 6 To understand single digit numbers</td>
</tr>
<tr>
<td>Lesson 7 To subtract a number from the same number</td>
</tr>
<tr>
<td>Lesson 8 To subtract one from a number</td>
</tr>
<tr>
<td>Lesson 9 To make familiar with all terms used in subtraction</td>
</tr>
</tbody>
</table>
Lesson 10 To learn that subtraction is the reverse process of addition
Lesson 11 To subtract a one digit number from a two digit without borrowing
Lesson 12 To subtract a one digit number from a two digit with borrowing
Lesson 13 To subtract a two digit number from a two digit without borrowing
Lesson 14 To subtract a two digit number from a two digit number with borrowing
Lesson 15 To subtract horizontally
Lesson 16 To subtract a three digit number from another three digit number without borrowing
Lesson 17 To subtract a three digit number from another three digit number with borrowing from ten’s place.
Lesson 18 To subtract a three digit number from another three digit number with borrowing from hundred’s place
Lesson 19 To subtract a three digit number from another three digit number with borrowing from both ten’s and hundred’s place
Lesson 20 To subtract one digit number from a three digit number
Lesson 21 To subtract two digit number from a three digit number
Lesson 22 To subtract any number from another number up to four digit numbers.

**Table 2: Lesson scheme for teaching addition**

**SPECIFIC OBJECTIVES OF EACH LESSON IN SEQUENTIAL ORDER**

Lesson 1 To learn the terms and symbol of addition
Lesson 2 To understand that addition means putting together things
Lesson 3 To make familiar with all terms used in addition
Lesson 4 To understand the concept of addition
Lesson 5 To understand the procedure of addition
Lesson 6 To add single digit numbers (sum not exceeding 10)
Lesson 7 To understand that when the order of the addends change, the sum remains the same
Lesson 8 To familiarize addition if doubtless
Lesson 9 To add one to a number
Lesson 10 To add zero to a number
Lesson 11 To add single digit number with another single digit number
Lesson 12 To add three or more single digit number
Lesson 13 To add a two digit number to a one digit number without carrying over
Lesson 14 To add a two digit number to another two digit number without carrying over
Lesson 15 To add a two digit number to another two digit number with carry over
Lesson 16 To add three or more groups of two digit numbers
Lesson 17 To add horizontally
Lesson 18 To add three or more groups of one and two digit
Lesson 19 To add a three digit number to a three digit number without carry over
Lesson 20 To add a three digit number to a three digit number with carry over the tens place
Lesson 21 To add a three digit number to a three digit number with carry over to hundred’s place
Lesson 22 To add a three digit number to a three digit number with carry over to hundred’s place
Lesson 23 To add three or more groups of three digit numbers
Lesson 24 To add three or more groups of four, three and two and one digit numbers
Lesson 25 To solve simple daily life problems involving addition

Phase III: Post test was administered after two weeks of completing the intervention teaching. Key math diagnostic arithmetic test, was given to participants of both experimental and control groups. Later, a master chart was
prepared and the data were entered to the computer for further computations.

Results and Discussion

Table 3 presents the means and standard deviation of both the groups in pre/post-tests for two mathematical operations viz., addition and subtraction. Data were analyzed using analysis of covariance (ANCOVA) with the pre-scores considered as covariates, and by ‘t’ test. ANCOVA revealed significant improvement in the addition test performance after remedial intervention for experimental group (F=23.40; p<.000). The mean pre and post-test scores of experimental group were found to be 5.75 and 10.55 respectively, whereas for the control group they were 6.25 and 6.75. Table 4 presents the summary of t-tests. There was no significant difference between experimental and control groups with respect to mean pre-test scores in addition (t = .907, p< .370). Also ANCOVA for subtraction showed significant improvement in the subtraction test performance after remedial intervention for experimental group (F=30.91; p<.001). The respective mean scores for pre and post-tests were 4.25 and 9.20, whereas the respective scores for the control group were 5.60 and 6.75. Table 4 shows that there was no significant difference between mean pre-intervention subtraction scores of experimental and control groups (t = 1.742, p< .091). Thus the significant improvement observed in the experiment group that underwent remedial teaching proved the effectiveness of the remedial program employed in the study. The results are in agreement with Gowramma (2005) and other researchers (Bahr & Rieth, 1989; Bolich, 1995; Chiang, 1986; Cook, Guzaukas, Pressley & Kerr, 1993; Cybriwsky & Scuster, 1990; Hasselbring, Goin & Bransford, 1988; Irish, 2002; Mcintry, Test, & Cooke, 1991; Mattingly & Bott, 1990; Morton & Flynt, 1997; Okolo, 1992; Ozaki, Williams, & Mclaughlin, 1996; Skinner, Beatty, Turco & Rasavage, 1989; Skinner, et. al., 1992; Stading, Williams & Mclaughlin, 1996; Van Houten & Rolider, 1990; VanLuit & Naglieri, 1999; Williams & Collins, 1994; Wilson & Majsterek, 1996; Wood, Frank & Wacker, 1998; Greene, 1999; Cook, & Reichard, 1996; Kroesbergen, VanLuit, & Naglieri, 2003).

Table 3: Group means and standard deviations in pre/post –tests for addition and subtraction

<table>
<thead>
<tr>
<th>Groups</th>
<th>Pre Test</th>
<th>Post Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>Ex/Addition</td>
<td>5.75</td>
<td>1.33</td>
</tr>
<tr>
<td>Co/Addition</td>
<td>6.25</td>
<td>2.07</td>
</tr>
<tr>
<td>Ex/Subtraction</td>
<td>4.25</td>
<td>1.29</td>
</tr>
<tr>
<td>Co/Subtraction</td>
<td>5.20</td>
<td>2.06</td>
</tr>
</tbody>
</table>

Table 4: Summary of “t” tests

<table>
<thead>
<tr>
<th>Test/operation</th>
<th>Experimental Group Mean</th>
<th>SD</th>
<th>Control Group Mean</th>
<th>SD</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test addition</td>
<td>5.75</td>
<td>1.33</td>
<td>6.25</td>
<td>2.07</td>
<td>.907</td>
</tr>
<tr>
<td>Pretest subtraction</td>
<td>4.25</td>
<td>1.29</td>
<td>5.20</td>
<td>2.06</td>
<td>1.742</td>
</tr>
</tbody>
</table>

Qualitative error analysis revealed the following pattern of errors in children with dyscalculia.

Addition:

A1: Not being aware of the basic knowledge of the addition process.

A2: Not being aware of the basic knowledge of the addition of fraction with the same denominator.

A3: Missing a digit (number) during the addition of digits in the same column.
A4: Not being able to infer as well as possible during the addition process.

A5: Not being able to add the digits (number) which appear horizontally.

A6: Not being aware of the concept of ‘zero’ in addition.

A7: Not being able to add decimal digit.

Table 5 presents the errors in addition performance along with an example and frequency of each particular error in both pre and post tests. One can notice that there was a drastic decrease in number of errors committed in almost all of the categories after the intervention.

<table>
<thead>
<tr>
<th>Errors</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>A6</th>
<th>A7</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>9</td>
<td>½+2/5=3/10</td>
<td>261</td>
<td>5+8=13</td>
<td>18+5=68</td>
<td>26</td>
<td>1.36</td>
<td></td>
</tr>
<tr>
<td>+6</td>
<td>40</td>
<td>5+18=23</td>
<td>+50</td>
<td>+2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>+751</td>
<td>5+28=?</td>
<td>70</td>
<td>1.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Pre test/f | 28 | 2 | 2 | 2 | 4 | 3 | 0 | 41 |
Post test/f | 5 | 1 | 0 | 2 | 2 | 1 | 3 | 14 |

Subtraction:

The qualitative analysis of errors in subtraction resulted in the following categories of errors:

S1: Not being aware of the basic knowledge of the subtraction process.

S2: Subtracting the units of the first digits (number) from the units of the second digits.

S3: Not being aware of the concept of ‘zero’ in subtraction.

S4: Not being aware of the basic knowledge of the subtraction of fraction with the same denominator.

S5: Not being aware of the concept of the additive identity.

S6: Not being aware of the concept of the ‘borrowing’

a) Not subtracting a digit from tens column after transmitting to the unit column.

b) Not subtracting a digit from hundreds column after transmitting to the tens column.

S7: Subtracting the units of tens.

S8: To leave the subtraction process incomplete.

Table 6 presents the errors in subtraction performance along with an example and frequency of each particular error in both pre and post tests. One can notice that there was a drastic decrease in number of errors committed in almost all of the categories after the intervention.

<table>
<thead>
<tr>
<th>Errors</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5-a</th>
<th>S5-b</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>5</td>
<td>62</td>
<td>500</td>
<td>7-2-5</td>
<td>73</td>
<td>217</td>
<td>32</td>
<td>73</td>
<td>5.7</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>-5</td>
<td>-304</td>
<td>9</td>
<td>9</td>
<td>0</td>
<td>-29</td>
<td>-32</td>
<td>-11</td>
<td>-22</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>63</td>
<td>204</td>
<td>54</td>
<td>285</td>
<td>22</td>
<td>1</td>
<td>5.3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Pre test/f | 29 | 8 | 8 | 2 | 8 | 7 | 2 | 6 | 0 | 70 |
Post test/f | 7 | 2 | 3 | 4 | 3 | 4 | 0 | 3 | 5 | 31 |
It may be noted that the errors under the categories A7 and S8 (A7 stands for addition of decimal digits and S8 stands for subtraction of decimal digits) increased in post test situation as compared to the pre-test condition. The reason behind such an unexpected performance of students was due to the fact that those topics were not covered in the classroom at the period of administering of pre test, and hence scored zero on them. Though these topics were taught in the class by the time of post-test, the students applied this knowledge wrongly. On the whole, it could be concluded that the remedial intervention improved significantly the basic knowledge of addition and subtraction.

References


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Alireza Beygi, Prakash Padakannaya, and Gowramma, I. P.


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Alireza Beygi, Department of Psychology, University of Mysore, Manasagangotri, Mysore - 570 006.

Gowramma, I. P, Department of Special Education, All India Institute of Speech and Hearing, Mysore - 570 006.

Prakash Padakannaya, PhD, Department of Psychology, University of Mysore, Manasagangotri, Mysore - 570 006. E-mail: prakashp99@yahoo.com, prakashp99@gmail.com
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